The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

MATHEMATICAL OLYMPIADS LECTURE NOTES

Vectors

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For our purposes, a *vector* is essentially a *directed arrow.* A *vector* has the properties of *length* and *direction,* but **not** *position.* Thus, we say two vectors are *equal* if they have the **same** *length* and **same** *direction.*

If A, B are two points then $\overrightarrow{AB}$ is the vector that joins A and B; it has *length* equal to the length of the line segment that joins A and B, and *direction* parallel to an arrow pointing from A to B. We assign coordinates to $\overrightarrow{AB}$ by subtracting the respective coordinates of A from those of B. The *position vector* of a point A is the vector $\overrightarrow{OA}$ where O is the origin, relative to which, A has coordinates. (So A and $\overrightarrow{OA}$ expressed as coordinates look the same.)

Example. *Suppose* $A = (1, 3)$ *and* $B = (2, 7)$ *then*

$$\overrightarrow{AB} = B - A = (2, 7) - (1, 3) = (2 - 1, 7 - 3) = (1, 4).$$

Suppose and $C = (5, 4)$ *and* $D = (6, 8)$ *then*

$$\overrightarrow{CD} = D - C = (6, 8) - (5, 4) = (6 - 5, 8 - 4) = (1, 4).$$

Observe that $\overrightarrow{AB}$ *and* $\overrightarrow{CD}$ *are* equal *despite the fact they connect different pairs of points in space.*