Fixed point free elements of prime order

Michael Giudici

University of Western Australia

Orbit-Counting Lemma

For a finite group G acting on a set Ω,

$$\# \text{ of orbits} = \frac{1}{|G|} \sum_{g \in G} |\operatorname{fix}(g)|$$

Orbit-Counting Lemma

For a finite group G acting on a set Ω,

$$\# \text{ of orbits} = \frac{1}{|G|} \sum_{g \in G} |\operatorname{fix}(g)|$$

If G is transitive then the average number of fixed points of elements of G is 1.

Orbit-Counting Lemma

For a finite group G acting on a set Ω,

$$\#\text{ of orbits} = \frac{1}{|G|}\sum_{g\in G} |\operatorname{fix}(g)|$$

If G is transitive then the average number of fixed points of elements of G is 1.

1_G has $|\Omega|$ fixed points.

Orbit-Counting Lemma

For a finite group G acting on a set Ω,

$$\# \text{ of orbits} = \frac{1}{|G|} \sum_{g \in G} |\operatorname{fix}(g)|$$

If G is transitive then the average number of fixed points of elements of G is 1.

1_G has $|\Omega|$ fixed points.

So if $|\Omega| > 1$, G must contain a fixed point free element.

How many?

Let $n = |\Omega| > 1, \quad G$ a transitive subgroup of $\mathrm{Sym}(\Omega)$.

$\delta(G) =$ proportion of fixed point free elements.

How many?

Let $n = |\Omega| > 1$, $\quad G$ a transitive subgroup of $\mathrm{Sym}(\Omega)$.

$\delta(G) =$ proportion of fixed point free elements.

Cameron-Cohen (1992): $\delta(G) \geq 1/n$ with equality if and only if G is sharply 2-transitive.

How many?

Let $n = |\Omega| > 1$, G a transitive subgroup of $\mathrm{Sym}(\Omega)$.

$\delta(G) =$ proportion of fixed point free elements.

Cameron-Cohen (1992): $\delta(G) \geq 1/n$ with equality if and only if G is sharply 2-transitive.

Guralnick-Wan (1997): If $n > 6$ then $\delta(G) > 2/n$ unless G a Frobenius group of order $n(n-1)/2$ or $n(n-1)$.

How many? II

Fulman-Guralnick (2003): There exists $\delta > 0$ such that $\delta(G) > \delta$ for all finite nonabelian simple groups G.

How many? II

Fulman-Guralnick (2003): There exists $\delta > 0$ such that $\delta(G) > \delta$ for all finite nonabelian simple groups G.

Fulman-Guralnick (2003): There exists $\delta > 0$ such that $\delta(G) > \delta/\log(n)$ for all primitive nonaffine groups G.

What are they?

Fein-Kantor-Schacher (1981): G has a fixed point free element of prime power order.

What are they?

Fein-Kantor-Schacher (1981): G has a fixed point free element of prime power order.

Generalised Isbell Conjecture

There is a function $f(p, k)$ such that if $n = p^a k$ with $p \not| k$ and $a \geq f(p, k)$ then G has a fixed point free element of p-power order.

Elusive groups

We say that a transitive permutation group is elusive if it has no fixed point free elements of prime order.

Elusive groups

We say that a transitive permutation group is elusive if it has no fixed point free elements of prime order.

For example, M_{11} acting on 12 points.

Elusive groups

We say that a transitive permutation group is elusive if it has no fixed point free elements of prime order.

For example, M_{11} acting on 12 points.

Existence of a fixed point free element of prime order is equivalent to existence of a semiregular subgroup.

Elusive groups

We say that a transitive permutation group is elusive if it has no fixed point free elements of prime order.

For example, M_{11} acting on 12 points.

Existence of a fixed point free element of prime order is equivalent to existence of a semiregular subgroup.

G is elusive on Ω if and only if every conjugacy class of elements of prime order meets G_ω nontrivially.

A question of Marušič

Marušič (1981): Are there any vertex-transitive digraphs with no fixed point free automorphisms of prime order?

A question of Marušič

Marušič (1981): Are there any vertex-transitive digraphs with no fixed point free automorphisms of prime order?

Independently posed by Jordan in 1988.

A question of Marušič

Marušič (1981): Are there any vertex-transitive digraphs with no fixed point free automorphisms of prime order?

Independently posed by Jordan in 1988.

A digraph is a Cayley digraph if and only if its automorphism group contains a regular subgroup.

A question of Marušič

Marušič (1981): Are there any vertex-transitive digraphs with no fixed point free automorphisms of prime order?

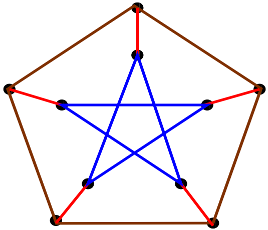
Independently posed by Jordan in 1988.

A digraph is a Cayley digraph if and only if its automorphism group contains a regular subgroup.

Many methods for constructing Hamiltonian cycles use existence of such an automorphism.

Nice representations

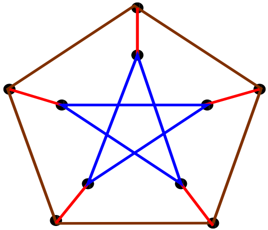
Biggs 1973

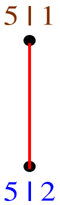


Petersen graph

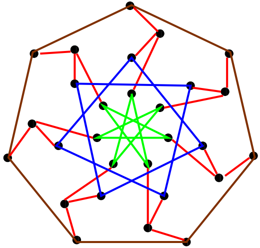
Nice representations

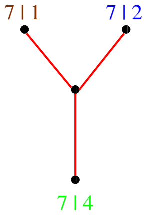
Biggs 1973





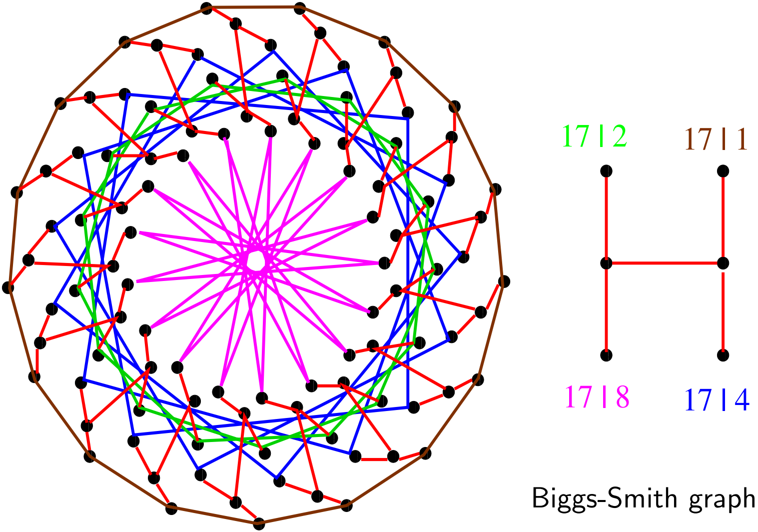
Petersen graph





Coxeter graph

Nice representations II



Biggs-Smith graph

2-closures

The 2-closure $G^{(2)}$ of G is the group of all permutations of Ω which fix setwise each orbit of G on $\Omega \times \Omega$.

2-closures

The 2-closure $G^{(2)}$ of G is the group of all permutations of Ω which fix setwise each orbit of G on $\Omega \times \Omega$.

If G is 2-transitive on Ω then $G^{(2)} = \mathrm{Sym}(\Omega)$.

2-closures

The 2-closure $G^{(2)}$ of G is the group of all permutations of Ω which fix setwise each orbit of G on $\Omega \times \Omega$.

If G is 2-transitive on Ω then $G^{(2)} = \mathrm{Sym}(\Omega)$.

$G^{(2)}$ preserves all systems of imprimitivity for G.

2-closures

The 2-closure $G^{(2)}$ of G is the group of all permutations of Ω which fix setwise each orbit of G on $\Omega \times \Omega$.

If G is 2-transitive on Ω then $G^{(2)} = \mathrm{Sym}(\Omega)$.

$G^{(2)}$ preserves all systems of imprimitivity for G.

We say that G is 2-closed if $G = G^{(2)}$.

2-closures

The 2-closure $G^{(2)}$ of G is the group of all permutations of Ω which fix setwise each orbit of G on $\Omega \times \Omega$.

If G is 2-transitive on Ω then $G^{(2)} = \mathrm{Sym}(\Omega)$.

$G^{(2)}$ preserves all systems of imprimitivity for G.

We say that G is 2-closed if $G = G^{(2)}$.

The full automorphism group of a digraph is 2-closed.

Polycirculant conjecture

Klin (1997) extended the question of Marušič to 2-closed groups.

Polycirculant conjecture

Klin (1997) extended the question of Marušič to 2-closed groups.

Polycirculant Conjecture

Every finite transitive 2-closed permutation group has a fixed point free element of prime order.

Polycirculant conjecture

Klin (1997) extended the question of Marušič to 2-closed groups.

Polycirculant Conjecture

Every finite transitive 2-closed permutation group has a fixed point free element of prime order.

In action on 12 points, $(M_{11})^{(2)} = S_{12}$.

Early results

Marušič (1981): All transitive permutation groups of degree p^k or mp, for some prime p and $m < p$, have a fixed point free element of order p.

Early results

Marušič (1981): All transitive permutation groups of degree p^k or mp, for some prime p and $m < p$, have a fixed point free element of order p.

Marušič and Scapellato (1993):

- All cubic vertex-transitive graphs have a fixed point free automorphism of prime order.

Early results

Marušič (1981): All transitive permutation groups of degree p^k or mp, for some prime p and $m < p$, have a fixed point free element of order p.

Marušič and Scapellato (1993):

- All cubic vertex-transitive graphs have a fixed point free automorphism of prime order.
- A vertex-transitive digraph of order $2p^2$ has a fixed point free automorphism of order p.

Fein-Kantor-Schacher examples

$\mathrm{AGL}(1, p^2)$, for p a Mersenne prime, acting on the set of $p(p+1)$ lines of the affine plane $\mathrm{AG}(2, p)$.

Fein-Kantor-Schacher examples

$\mathrm{AGL}(1, p^2)$, for p a Mersenne prime, acting on the set of $p(p+1)$ lines of the affine plane $\mathrm{AG}(2, p)$.

All elements of order 2 and p fix a line so action is elusive.

Fein-Kantor-Schacher examples

$\mathrm{AGL}(1, p^2)$, for p a Mersenne prime, acting on the set of $p(p+1)$ lines of the affine plane $\mathrm{AG}(2, p)$.

All elements of order 2 and p fix a line so action is elusive.

2-closure contains C_p^{p+1} whose $p+1$ orbits are the parallel classes.

Fein-Kantor-Schacher examples

$\mathrm{AGL}(1, p^2)$, for p a Mersenne prime, acting on the set of $p(p+1)$ lines of the affine plane $\mathrm{AG}(2, p)$.

All elements of order 2 and p fix a line so action is elusive.

2-closure contains C_p^{p+1} whose $p+1$ orbits are the parallel classes.

$\mathrm{A\Gamma L}(1, p^2)$ is also elusive in this action.

More constructions

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

Suppose G_1, G_2 are elusive groups on the sets Ω_1, Ω_2 respectively.

More constructions

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

Suppose G_1, G_2 are elusive groups on the sets Ω_1, Ω_2 respectively.

Then

- $G_1 \times G_2$ is elusive on $\Omega_1 \times \Omega_2$.

More constructions

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

Suppose G_1, G_2 are elusive groups on the sets Ω_1, Ω_2 respectively.

Then

- $G_1 \times G_2$ is elusive on $\Omega_1 \times \Omega_2$.
- $G_1 \text{ wr } G_2$ is elusive on $\Omega_1 \times \Omega_2$.

More constructions

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

Suppose G_1, G_2 are elusive groups on the sets Ω_1, Ω_2 respectively.

Then

- $G_1 \times G_2$ is elusive on $\Omega_1 \times \Omega_2$.
- $G_1 \operatorname{wr} G_2$ is elusive on $\Omega_1 \times \Omega_2$.
- $G_1 \operatorname{wr} S_n$ is elusive on Ω^n.

General affine construction

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

- V a vector space over a field of characteristic p,
- $G_1 \leqslant \mathrm{GL}(V)$ with order prime to p,
- W a subspace of V,
- $H_1 < G_1$ fixes W setwise.

General affine construction

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

- V a vector space over a field of characteristic p,
- $G_1 \leqslant \mathrm{GL}(V)$ with order prime to p,
- W a subspace of V,
- $H_1 < G_1$ fixes W setwise.

The action of $V \rtimes G_1$ on the set of right cosets of $W \rtimes H_1$ is elusive if and only if

1. the images of W under G_1 cover V, and
2. every conjugacy class of elements of prime order in G_1 meets H_1.

FKS examples again

$$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$$

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

$G_1 = C_{p^2-1} \leqslant \mathrm{GL}(2, p)$, W a 1-dimensional subspace, $H_1 = C_{p-1}$

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

$G_1 = C_{p^2-1} \leqslant \mathrm{GL}(2, p)$, W a 1-dimensional subspace, $H_1 = C_{p-1}$

1. G_1 acts transitively on nontrivial elements of V.

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

$G_1 = C_{p^2-1} \leqslant \mathrm{GL}(2, p)$, W a 1-dimensional subspace, $H_1 = C_{p-1}$

1. G_1 acts transitively on nontrivial elements of V.
2. Only primes dividing $p^2 - 1$ are those dividing $p - 1$.

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

$G_1 = C_{p^2-1} \leqslant \mathrm{GL}(2, p)$, W a 1-dimensional subspace, $H_1 = C_{p-1}$

1. G_1 acts transitively on nontrivial elements of V.
2. Only primes dividing $p^2 - 1$ are those dividing $p - 1$.

✓

FKS examples again

$\mathrm{AGL}(1, p^2) \cong C_p^2 \rtimes C_{p^2-1}$

Stabiliser of a line is isomorphic to $C_p \rtimes C_{p-1}$

$G_1 = C_{p^2-1} \leqslant \mathrm{GL}(2, p)$, W a 1-dimensional subspace, $H_1 = C_{p-1}$

1. G_1 acts transitively on nontrivial elements of V.
2. Only primes dividing $p^2 - 1$ are those dividing $p - 1$.

In fact, any group $C_p^2 \rtimes G_1$, where $G_1 \leqslant C_{p^2-1}$ contains the Sylow 2-subgroup, is elusive.

Some more affine examples

- $V = \mathrm{GF}(7^3)$, W a 2-dimensional subspace, $G_1 = 3^{1+2} \rtimes Q_8$, $H_1 = 3^2 \rtimes C_2$. (CGJKLMN 2002)

Some more affine examples

- $V = \mathrm{GF}(7^3)$, W a 2-dimensional subspace, $G_1 = 3^{1+2} \rtimes Q_8$, $H_1 = 3^2 \rtimes C_2$. (CGJKLMN 2002)

- p a Mersenne prime, $V = \mathrm{GF}(2)^{p^2-1}$, W a hyperplane, $G_1 = \mathrm{A\Gamma L}(1, p^2)$, $H_1 = (C_p \rtimes C_{p-1}) \times C_2$. (MG 2007)

Some more affine examples

- $V = \mathrm{GF}(7^3)$, W a 2-dimensional subspace, $G_1 = 3^{1+2} \rtimes Q_8$, $H_1 = 3^2 \rtimes C_2$. (CGJKLMN 2002)
- p a Mersenne prime, $V = \mathrm{GF}(2)^{p^2-1}$, W a hyperplane, $G_1 = \mathrm{A\Gamma L}(1, p^2)$, $H_1 = (C_p \rtimes C_{p-1}) \times C_2$. (MG 2007)
- $V = \mathrm{GF}(2)^8$, W a codimension 2 space, $G_1 = \mathrm{A\Gamma L}(1, 9)$, $H_1 = (C_3 \rtimes C_2) \times C_2$. (MG 2007)

Hensel lifting construction

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

- V, G_1, W and H_1 as in affine construction.
- V dimension d over $\mathrm{GF}(p)$.
- W dimension k.

Hensel lifting construction

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

- V, G_1, W and H_1 as in affine construction.
- V dimension d over $\mathrm{GF}(p)$.
- W dimension k.

Then for any $m > 1$, the action of $(C_{p^m})^d \rtimes G_1$ on the set of cosets of $(C_{p^m})^k \rtimes H_1$ is elusive of degree $p^{km}|G_1 : H_1|$.

Hensel lifting construction

Cameron-MG-Jones-Kantor-Klin-Marušič-Nowitz (2002)

- V, G_1, W and H_1 as in affine construction.
- V dimension d over $\mathrm{GF}(p)$.
- W dimension k.

Then for any $m > 1$, the action of $(C_{p^m})^d \rtimes G_1$ on the set of cosets of $(C_{p^m})^k \rtimes H_1$ is elusive of degree $p^{km}|G_1 : H_1|$.

This takes elusive groups of degree d and builds new ones with degree $p^m d$.

Priming Construction

MG 2007

Input:

- $V \rtimes \langle a \rangle$ elusive of degree n,
- $\langle a \rangle$ irreducible on V with order prime to p,
- point stabiliser $W \rtimes \langle b \rangle$,
- r a prime dividing $|a|$.

Priming Construction

MG 2007

Input:

- $V \rtimes \langle a \rangle$ elusive of degree n,
- $\langle a \rangle$ irreducible on V with order prime to p,
- point stabiliser $W \rtimes \langle b \rangle$,
- r a prime dividing $|a|$.

Let

- $V' = \underbrace{V \oplus \cdots \oplus V}_{r \text{ copies}}, \qquad W' = W \oplus V \oplus \cdots \oplus V$
- $g = (1, \ldots, 1, a)\tau$ where $\tau = (12 \ldots r)$

Priming Construction

MG 2007

Input:

- $V \rtimes \langle a \rangle$ elusive of degree n,
- $\langle a \rangle$ irreducible on V with order prime to p,
- point stabiliser $W \rtimes \langle b \rangle$,
- r a prime dividing $|a|$.

Let

- $V' = \underbrace{V \oplus \cdots \oplus V}_{r \text{ copies}}$, $\quad W' = W \oplus V \oplus \cdots \oplus V$
- $g = (1, \ldots, 1, a)\tau$ where $\tau = (12 \ldots r)$

Then $V' \rtimes \langle g \rangle$ acts elusively on the set of right cosets of $W' \rtimes \langle (b, \ldots, b) \rangle$ with degree rn.

Priming Construction II

Using the FKS examples as a starting point, we get elusive groups for all degrees

$$p2^n r_1^{j_1} \dots r_t^{j_t}$$

where

- p Mersenne,
- $2^n > p$,
- $r_1, \dots, r_t$ are distinct odd primes dividing $p - 1$,
- $j_i \geq 0$.

Priming Construction II

Using the FKS examples as a starting point, we get elusive groups for all degrees

$$p2^n r_1^{j_1} \dots r_t^{j_t}$$

where

- p Mersenne,
- $2^n > p$,
- $r_1, \dots, r_t$ are distinct odd primes dividing $p - 1$,
- $j_i \geq 0$.

Using Hensel lifting construction can also multiply by powers of p.

A characterisation

Theorem (MG-Kelly 2008+)

Let G be an elusive permutation group such that $G = N \rtimes G_1$ for N an elementary abelian minimal normal subgroup and G_1 cyclic. Then G can be obtained by repeatedly applying the priming construction to an FKS-group.

Primitive groups and generalisations

Let $G \leqslant \mathrm{Sym}(\Omega)$ transitive

- G is primitive if there are no nontrivial partitions of Ω preserved by G.

Primitive groups and generalisations

Let $G \leqslant \mathrm{Sym}(\Omega)$ transitive

- G is primitive if there are no nontrivial partitions of Ω preserved by G.
- G is quasiprimitive if all nontrivial normal subgroup of G are transitive.

Primitive groups and generalisations

Let $G \leqslant \mathrm{Sym}(\Omega)$ transitive

- G is primitive if there are no nontrivial partitions of Ω preserved by G.
- G is quasiprimitive if all nontrivial normal subgroup of G are transitive.
- G is biquasiprimitive if G is not quasiprimitive and all nontrivial normal subgroups have at most two orbits.

Classifications

Theorem (MG 2003)

The only almost simple elusive groups are M_{11} *and* $M_{10} = A_6 \cdot 2$ *acting on 12 points.*

Classifications

Theorem (MG 2003)

The only almost simple elusive groups are M_{11} *and* $M_{10} = A_6 \cdot 2$ *acting on 12 points.*

Theorem (MG 2003)

Let G *be an elusive permutation group with a transitive minimal normal subgroup. Then* $G = M_{11} \operatorname{wr} K$ *acting on* 12^k *points with* K *a transitive subgroup of* S_k.

Classifications

Theorem (MG 2003)

The only almost simple elusive groups are M_{11} *and* $M_{10} = A_6 \cdot 2$ *acting on 12 points.*

Theorem (MG 2003)

Let G *be an elusive permutation group with a transitive minimal normal subgroup. Then* $G = M_{11} \operatorname{wr} K$ *acting on* 12^k *points with* K *a transitive subgroup of* S_k*.*

2-closures of exceptions contain S_{12}^k.

Classifications

Theorem (MG 2003)

The only almost simple elusive groups are M_{11} and $M_{10} = A_6 \cdot 2$ acting on 12 points.

Theorem (MG 2003)

Let G be an elusive permutation group with a transitive minimal normal subgroup. Then $G = M_{11} \operatorname{wr} K$ acting on 12^k points with K a transitive subgroup of S_k.

2-closures of exceptions contain S_{12}^k.

All minimal normal subgroups of a counterexample to the polycirculant conjecture must be intransitive.

Classifications II

Theorem (MG-Xu 2007)

Let G be a biquasiprimitive elusive permutation group on Ω. Then one of the following holds:

1. $G = M_{10}$ *and* $|\Omega| = 12$;
2. $G = M_{11} \operatorname{wr} K$ *and* $|\Omega| = 2(12^k)$, *where* $K \leqslant S_k$ *is transitive with an index two subgroup;*
3. $G = M_{11} \operatorname{wr} K$ *and* $|\Omega| = 2(12)^{k/2}$, *where* $K \leqslant S_k$ *is transitive with an index two intransitive subgroup.*

Classifications II

Theorem (MG-Xu 2007)

Let G be a biquasiprimitive elusive permutation group on Ω. Then one of the following holds:

1. $G = M_{10}$ *and* $|\Omega| = 12$*;*
2. $G = M_{11} \,\mathrm{wr}\, K$ *and* $|\Omega| = 2(12^k)$*, where* $K \leqslant S_k$ *is transitive with an index two subgroup;*
3. $G = M_{11} \,\mathrm{wr}\, K$ *and* $|\Omega| = 2(12)^{k/2}$*, where* $K \leqslant S_k$ *is transitive with an index two intransitive subgroup.*

Corollary

Let G be a biquasiprimitive elusive permutation group on Ω. Then $G^{(2)}$ *is not elusive.*

Locally quasiprimitive graphs

Γ a graph, $G \leqslant \mathrm{Aut}(\Gamma)$.

Locally quasiprimitive graphs

Γ a graph, $G \leqslant \mathrm{Aut}(\Gamma)$.

Γ is G-locally quasiprimitive if for all vertices v, G_v is quasiprimitive on $\Gamma(v)$.

Locally quasiprimitive graphs

Γ a graph, $G \leqslant \mathrm{Aut}(\Gamma)$.

Γ is G-locally quasiprimitive if for all vertices v, G_v is quasiprimitive on $\Gamma(v)$.

A 2-arc in a graph is a triple (v_0, v_1, v_2) such that $v_0 \sim v_1 \sim v_2$ and $v_0 \neq v_2$.

Locally quasiprimitive graphs

Γ a graph, $G \leqslant \mathrm{Aut}(\Gamma)$.

Γ is *G-locally quasiprimitive* if for all vertices v, G_v is quasiprimitive on $\Gamma(v)$.

A *2-arc* in a graph is a triple (v_0, v_1, v_2) such that $v_0 \sim v_1 \sim v_2$ and $v_0 \neq v_2$.

Γ is *2-arc transitive* if $\mathrm{Aut}(\Gamma)$ is transitive on the set of 2-arcs in Γ.

Locally quasiprimitive graphs

Γ a graph, $G \leqslant \mathrm{Aut}(\Gamma)$.

Γ is G-locally quasiprimitive if for all vertices v, G_v is quasiprimitive on $\Gamma(v)$.

A 2-arc in a graph is a triple (v_0, v_1, v_2) such that $v_0 \sim v_1 \sim v_2$ and $v_0 \neq v_2$.

Γ is 2-arc transitive if $\mathrm{Aut}(\Gamma)$ is transitive on the set of 2-arcs in Γ.

A vertex-transitive graph is 2-arc-transitive if and only if G_v is 2-transitive on $\Gamma(v)$ for all v.

Locally quasiprimitive graphs II

Theorem (Praeger 1985)

Let Γ be a finite connected G-vertex-transitive, G-locally-quasiprimitive graph and let $1 \neq N \lhd G$. If N has at least three vertex-orbits then it is semiregular.

Locally quasiprimitive graphs II

Theorem (Praeger 1985)

Let Γ be a finite connected G-vertex-transitive, G-locally-quasiprimitive graph and let $1 \neq N \lhd G$. If N has at least three vertex-orbits then it is semiregular.

Corollary

Either G contains a semiregular subgroup, or G is quasiprimitive or biquasiprimitive on vertices.

Locally quasiprimitive graphs III

Theorem (MG-Xu 2007)

Every vertex-transitive, locally quasiprimitive graph has a fixed point free automorphism of prime order.

Locally quasiprimitive graphs III

Theorem (MG-Xu 2007)

Every vertex-transitive, locally quasiprimitive graph has a fixed point free automorphism of prime order.

Corollary

Every 2-arc-transitive graph has a fixed point free automorphism of prime order.

Locally quasiprimitive graphs III

Theorem (MG-Xu 2007)

Every vertex-transitive, locally quasiprimitive graph has a fixed point free automorphism of prime order.

Corollary

Every 2-arc-transitive graph has a fixed point free automorphism of prime order.

Corollary

Every arc-transitive graph of prime valency has a fixed point free automorphism of prime order.

Some more results

Dobson-Malnič-Marušič-Nowitz (2007):

- All quartic vertex-transitive graphs have a fixed point free automorphism of prime order.

Some more results

Dobson-Malnič-Marušič-Nowitz (2007):

- All quartic vertex-transitive graphs have a fixed point free automorphism of prime order.
- All vertex-transitive graphs of valency $p+1$ admitting a transitive $\{2, p\}$-group for p odd have a fixed point free automorphism of prime order.

Some more results

Dobson-Malnič-Marušič-Nowitz (2007):

- All quartic vertex-transitive graphs have a fixed point free automorphism of prime order.
- All vertex-transitive graphs of valency $p+1$ admitting a transitive $\{2, p\}$-group for p odd have a fixed point free automorphism of prime order.
- There are no elusive 2-closed groups of square-free degree.

Some more results

Dobson-Malnič-Marušič-Nowitz (2007):

- All quartic vertex-transitive graphs have a fixed point free automorphism of prime order.
- All vertex-transitive graphs of valency $p+1$ admitting a transitive $\{2,p\}$-group for p odd have a fixed point free automorphism of prime order.
- There are no elusive 2-closed groups of square-free degree.

Xu (2008): All arc-transitive graphs with valency pq, p,q primes, such that $\mathrm{Aut}(\Gamma)$ has a nonabelian minimal normal subgroup N with at least 3 vertex orbits, has a semiregular automorphism.

Open problems and questions

- Prove the polycirculant conjecture.

Open problems and questions

- Prove the polycirculant conjecture.
- Prove the polycirculant conjecture for arc-transitive graphs.

Open problems and questions

- Prove the polycirculant conjecture.
- Prove the polycirculant conjecture for arc-transitive graphs.
- Find new constructions of elusive groups.

Open problems and questions

- Prove the polycirculant conjecture.
- Prove the polycirculant conjecture for arc-transitive graphs.
- Find new constructions of elusive groups.
- For what degrees do elusive groups exist? (smallest degree for which existence is unknown is 40.)

Open problems and questions

- Prove the polycirculant conjecture.
- Prove the polycirculant conjecture for arc-transitive graphs.
- Find new constructions of elusive groups.
- For what degrees do elusive groups exist? (smallest degree for which existence is unknown is 40.)
- Does the set of all such degrees have density 0?

Open problems and questions

- Prove the polycirculant conjecture.
- Prove the polycirculant conjecture for arc-transitive graphs.
- Find new constructions of elusive groups.
- For what degrees do elusive groups exist? (smallest degree for which existence is unknown is 40.)
- Does the set of all such degrees have density 0?

`http://www.maths.uwa.edu.au/~giudici/research.html`

Group Theory, Combinatorics and Computation

AMSI theme program at The University of Western Australia

$5^{\mathrm{th}} - 16^{\mathrm{th}}$ January 2009

First week international conference in honour of Cheryl Praeger's 60^{th} birthday.

Informal second week consisting of short courses and problem sessions.

For more information, see

`http://sponsored.uwa.edu.au/gcc09/welcome`