

# Solutions for Workshop

## Necklaces and Chemicals

Alice Niemeyer

1. Note that conjugate permutations have the same cycle structure.

(a)  $G = S_5$ ,

```
gap> grp := SymmetricGroup(5);;
gap> cc := ConjugacyClasses(grp);
[ ()^G, (1,2)^G, (1,2)(3,4)^G, (1,2,3)^G,
(1,2,3)(4,5)^G, (1,2,3,4)^G, (1,2,3,4,5)^G ]
```

Hence elements with the same cycle structure are conjugate in  $S_5$ .

(b)  $G = A_5$ ,

```
gap> grp := AlternatingGroup(5);;
gap> cc := ConjugacyClasses(grp);
[ ()^G, (1,2)(3,4)^G, (1,2,3)^G,
(1,2,3,4,5)^G, (1,2,3,5,4)^G ]
```

The elements  $(1,2,3,4,5)$  and  $(1,2,3,5,4)$  have the same cycle structure but are not conjugate in  $A_5$ .

(c)  $G = D_{10}$

```
gap> grp := DihedralGroup(IsPermGroup, 10 );;
gap> cc := ConjugacyClasses(grp);
[ ()^G, (2,5)(3,4)^G, (1,2,3,4,5)^G, (1,3,5,2,4)^G ]
```

The elements  $(1,2,3,4,5)$  and  $(1,3,5,2,4)$  have the same cycle structure but are not conjugate in  $D_{10}$ . Thus the rotations by  $72^\circ$  degrees and  $36^\circ$  degrees are not conjugate.

2. See the accompanying GAP function `NrCyclesPerm()`.

3. See the accompanying GAP function `NrColourings()`.

4. `gap> NrColourings( DihedralGroup( IsPermGroup, 12 ), 3 );`  
92

5. `gap> NrColourings( DihedralGroup( IsPermGroup, 18 ), 3 );`  
1219