Workshop on GAP and on Permutation Groups

Alice C. Niemeyer

Programming and Groups

A permutation on n points can be written in disjoint cycle notation.
For example the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 7 & 6 & 4 \end{pmatrix} \in S_7$$

can be expressed as

$$(1, 2, 3)(4, 5, 7)$$

in disjoint cycle notation.
Permutations can either be typed into GAP in disjoint cycle notation, e.g.

```
gap> g := (1,2,3)(4,5, 7);
```

or as lists in second row notation and converted to Permutations.

```
gap> g := PermList( [ 2,3,1,5,7,6,4] );
```

The GAP-function `CycleStructurePerm` determines the cycle structure of a permutation.

```
gap> CycleStructurePerm(g);
```

$[, 2]$
The above command returns a list s such that

$$s[i] = \begin{cases} \text{empty} & g \text{ has no cycles of length } i \\ j & g \text{ has } j \text{ cycles of length } i + 1 \end{cases}$$

The function `NrMovedPoints()` counts how many points are moved by a permutation or a permutation group and `MovedPoints()` returns a list of all points moved by a permutation or a permutation group.
The function `Elements()` can be used to generate a list of all elements of a group.

Problems

1. Loop over the list of elements of M_{11} to test whether there is an element of order 10.

2. Use the function `Filtered` to find all elements of order 5 in M_{11}.

3. Among those, can you find one that maps 2 to 5 and 5 to 8?

4. Write a GAP function which counts the number of elements of each order in a group. How many elements of each order are there in

   (a) D_{12}?

   (b) M_{11}?

   (c) S_6?

5. Write a GAP function which takes as input an element g and a permutation group G such that $g \in G$. The function counts the number of elements in G which have the same cycle structure as g.

6. Are all elements of the same cycle structure conjugate in

   (a) D_{12}?

   (b) M_{11}?

   (c) S_6?

   Use the GAP-function `ConjugacyClasses()`.

7. The function `Stabilizer(G,a)` computes the stabiliser of a in the group G. Find basis and compute generators for the stabilisers in the corresponding stabiliser chain for the following groups:

   (a) $G_1 = \langle(1,2)(3,4,5,6)\rangle$,

   (b) $G_2 = D_{2n}$ for $n = 7, 8$,

   (c) MathieuGroup M_{11} of degree 11,

   (d) S_n for $n = 5, 6$.

8. Find the order of M_{11} in the previous exercise using the Orbit-Stabiliser Theorem. Note that `Orbit(G, a)` computes the orbit of the point `a` under G. Compare the values you computed with the values returned by `Size(G)`.

9. The GAP command `in` test whether an element lies in a group, e.g. `(1,2) in SymmetricGroup(3)` returns `true`. Use this command to show that M_{11} is a proper subgroup of S_{11} by finding a permutation in S_{11} which does not lie in M_{11}.

10. Express the permutation $(1, 10, 4, 3, 6, 9)(2, 8)(5, 7, 11)$ in the generators of M_{11} using the GAP-function `Factorisation()`.